Integrated Mathematics I

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Integrated Mathematics I:

Standard 1 — Number Sense and Computation

Students deepen their understanding of real numbers by comparing expressions involving square roots and exponents. They use the properties of real numbers to simplify algebraic formulas and they use dimensional analysis to convert between different measurement units.

Standard 2 — Algebra and Functions

Students solve and graph linear equations and inequalities using order properties of the real numbers and they solve word problems involving linear equations, inequalities, and formulas. Students solve pairs of linear equations in two variables using both graphs and algebraic methods. Students operate with polynomials — adding, subtracting, multiplying, dividing, and raising to powers. They draw graphs of quadratic, cubic, and rational functions and solve problems of growth and decay.

Standard 3 — Geometry and Measurement

Students identify and describe polygons (triangles, quadrilaterals, pentagons, hexagons, etc.), using terms such as regular, convex, and concave. They find measures of sides, perimeters, and areas of polygons, justifying their methods, and they apply transformations to polygons. Students prove the Pythagorean Theorem and apply it to solving problems. They explore relationships among the faces, edges, and vertices of polyhedra and describe symmetries of solids.

Standard 4 — Data Analysis and Statistics

Students will use and analyze a variety of data displays: line plots, histograms, stem-and-leaf plots, frequency tables, and scatterplots. They will understand a number of measures of central tendency and variability, as well as recognize patterns in tables and graphs of linear data.

Standard 5 — Probability

Students will use simulations, find empirical and theoretical probabilities, and use the Law of Large Numbers. They will understand independent events and probability distributions.

Standard 6 — Discrete Mathematics

Students will construct vertex-edge graphs and digraphs and will use Euler paths and recursion equations to solve problems. They will use matrices to describe vertex-edge graphs and they will find row and column sums for matrices.

Standard 7 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. They also learn about inductive and deductive reasoning and how to use counterexamples to show that a general statement is false.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, $\frac{1}{4}$, one divided by four, 0.25, $\frac{1}{8} + \frac{1}{8}$, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Number Sense and Computation

Students simplify and compare expressions. They use rational exponents and simplify square roots.

IM1.1.1 Compare real number expressions.

Example: Which is larger: 2^3 or $\sqrt{49}$?

IM1.1.2 Simplify square roots using factors.

Example: Explain why $\sqrt{48} = 4\sqrt{3}$.

IM1.1.3 Understand and use the distributive, associative, and commutative properties.

Example: Simplify $(6x^2 - 5x + 1) - 2(x^2 + 3x - 4)$ by removing the parentheses and rearranging. Explain why you can carry out each step.

IM1.1.4 Use the laws of exponents for rational exponents.

Example: Simplify $25^{3/2}$.

IM1.1.5 Use dimensional (unit) analysis to organize conversions and computations.

Example: Convert 5 miles per hour to feet per second: $\frac{5 \text{ mi}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \approx 7.3$ ft/sec.

Standard 2

Algebra and Functions

Students solve linear equations and inequalities in one variable. They write equations of lines and find and use the slope and y-intercept of lines. Students solve pairs of linear equations using graphs and algebra. Students add, subtract, multiply, and divide polynomials and solve word problems using exponential functions.

IM1.2.1 Solve linear equations.

Example: Solve the equation $7a + 2 = 5a - 3a + 8$.

IM1.2.2 Solve equations and formulas for a specified variable.

Example: Solve the equation $q = 4p - 11$ for p.

IM1.2.3 Find solution sets of linear inequalities when possible numbers are given for the variable.

Example: Solve the inequality $6x - 3 > 10$ for x in the set $\{0, 1, 2, 3, 4\}$.

IM1.2.4 Solve linear inequalities using properties of order.

Example: Solve the inequality $8x - 7 \leq 2x + 5$, explaining each step in your solution.

IM1.2.5 Solve word problems that involve linear equations, formulas, and inequalities.

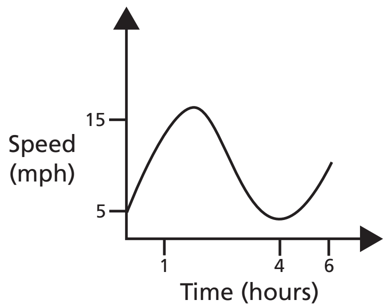
Example: You are selling tickets for a play that cost \$3 each. You want to sell at least \$50 worth. Write and solve an inequality for the number of tickets you must sell.

IM1.2.6 Sketch a reasonable graph for a given relationship.

Example: Sketch a reasonable graph for a person's height from age 0 to 25.

IM1.2.7 Interpret a graph representing a given situation.

Example: Jessica is riding a bicycle. The graph below shows her speed as it relates to the time she has spent riding. Describe what might have happened to account for such a graph.



IM1.2.8 Understand the concept of a function, decide if a given relation is a function, and link equations to functions.

Example: Use either paper or a spreadsheet to generate a list of values for x and y in $y = x^2$. Based on your data, make a conjecture about whether or not this relation is a function. Explain your reasoning.

IM1.2.9 Find the domain and range of a relation.

Example: Based on the list of values from the example in indicator 2.8, what is the domain and range of $y = x^2$?

IM1.2.10 Graph a linear equation.

Example: Graph the equation $3x - y = 2$.

IM1.2.11 Find the slope, x-intercept, and y-intercept of a line given its graph, its equation, or two points on the line.

Example: Find the slope and y-intercept of the line $4x + 6y = 12$.

IM1.2.12 Write the equation of a line in slope-intercept form. Understand how the slope and y-intercept of the graph are related to the equation.

Example: Write the equation of the line $4x + 6y = 12$ in slope-intercept form. What is the slope of this line? Explain your answer.

IM1.2.13 Write the equation of a line given appropriate information.

Example: Find an equation of the line through the points $(1, 4)$ and $(3, 10)$, then find an equation of the line through the point $(1, 4)$ perpendicular to the first line.

IM1.2.14 Write the equation of a line that models a data set and use the equation (or the graph of the equation) to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change.

Example: As your family is traveling along an interstate, you note the distance traveled every 5 minutes. A graph of time and distance shows that the relation is approximately linear. Write the equation of the line that fits your data. Predict the time for a journey of 50 miles. What does the slope represent?

IM1.2.15 Use a graph to estimate the solution of a pair of linear equations in two variables.

Example: Graph the equations $3y - x = 0$ and $2x + 4y = 15$ to find where the lines intersect.

IM1.2.16 Understand and use the substitution method to solve a pair of linear equations in two variables.

Example: Solve the equations $y = 2x$ and $2x + 3y = 12$ by substitution.

IM1.2.17 Understand and use the addition or subtraction method to solve a pair of linear equations in two variables.

Example: Use subtraction to solve the equations: $3x + 4y = 11$ and $3x + 2y = 7$.

IM1.2.18 Understand and use multiplication with the addition or subtraction method to solve a pair of linear equations in two variables.

Example: Use multiplication with the subtraction method to solve the equations: $x + 4y = 16$ and $3x + 2y = -3$.

IM1.2.19 Use pairs of linear equations to solve word problems.

Example: The income a company makes from a certain product can be represented by the equation $y = 10.5x$ and the expenses for that product can be represented by the equation $y = 5.25x + 10{,}000$, where x is the amount of the product sold and y is the number of dollars. How much of the product must be sold for the company to reach the break-even point?

IM1.2.20 Add and subtract polynomials.

Example: Simplify $(4x^2 - 7x + 2) - (x^2 + 4x - 5)$.

IM1.2.21 Multiply and divide monomials.

Example: Simplify $a^2b^5 \div ab^2$.

IM1.2.22 Find powers and roots of monomials (only when the answer has an integer exponent).

Example: Find the square root of a^2b^6.

IM1.2.23 Multiply polynomials.

Example: Multiply $(n + 2)(4n - 5)$.

IM1.2.24 Divide polynomials by monomials.

Example: Divide $4x^3y^2 + 8xy^4 - 6x^2y^5$ by $2xy^2$.

IM1.2.25 Understand and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression.

Example: A graphing calculator can be used to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth. Justify using the x-intercepts of $y = 3x^2 - 5x - 1$ as the solutions of the equation.

IM1.2.26 Graph quadratic, cubic, and radical equations.

Example: Draw the graph of $y = x^2 - 3x + 2$. Using a graphing calculator or a spreadsheet (generate a data set), display the graph to check your work.

IM1.2.27 Solve quadratic equations using the quadratic formula.

Example: Solve the equation $x^2 - 7x + 9 = 0$.

IM1.2.28 **Use quadratic equations to solve word problems.**

Example: A ball falls so that its distance above the ground can be modeled by the equation $s = 100 - 16t^2$, where s is the distance above the ground in feet and t is the time in seconds. According to this model, at what time does the ball hit the ground?

IM1.2.29 **Use graphing technology to find approximate solutions of quadratic and cubic equations.**

Example: Use a graphing calculator to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth.

IM1.2.30 **Graph exponential functions.**

Example: Draw the graphs of the functions $y = 2^x$ and $y = 2^{-x}$.

IM1.2.31 **Solve word problems involving applications of exponential functions to growth and decay.**

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million, and 400 million. What do you notice about these time periods?

Standard 3

Geometry and Measurement

Students identify and describe polygons, including finding measures of sides, perimeters, and areas. They use congruence, similarity, symmetry, tessellations, and transformations. Students understand the Pythagorean Theorem and use it to solve problems. They describe relationships and symmetries in geometric solids.

IM1.3.1 **Identify and describe convex, concave, and regular polygons.**

Example: Draw a regular hexagon. Is it convex or concave? Explain your answer.

IM1.3.2 **Apply transformations (slides, flips, turns, expansions, and contractions) to polygons to determine congruence, similarity, symmetry, and tessellations. Know that images formed by slides, flips, and turns are congruent to the original shape.**

Example: Use a drawing program to create regular hexagons, regular octagons, and regular pentagons. Under the drawings, describe which of the polygons would be best for tiling a rectangular floor. Explain your reasoning.

IM1.3.3 **Find and use measures of sides, perimeters, and areas of polygons. Relate these measures to each other using formulas.**

Example: A rectangle of area 360 square yards is 10 times as long as it is wide. Find its length and width.

IM1.3.4 **Use properties of congruent and similar quadrilaterals to solve problems involving lengths and areas.**

Example: Of two similar rectangles, the second has sides three times the length of the first. How many times larger in area is the second rectangle?

IM1.3.5 Find and use measures of sides, perimeters, and areas of quadrilaterals. Relate these measures to each other using formulas.

> **Example:** A section of roof is a trapezoid with length 4 m at the ridge and 6 m at the gutter. The shortest distance from ridge to gutter is 3 m. Construct a model using a drawing program, showing how to find the area of this section of roof.

IM1.3.6 Prove and use the Pythagorean Theorem.

> **Example:** On each side of a right triangle, draw a square with that side of the triangle as one side of the square. Find the areas of the three squares. What relationship is there between the areas?

IM1.3.7 Describe relationships between the faces, edges, and vertices of polyhedra.

> **Example:** Count the sides, edges, and corners of a square-based pyramid. How are these numbers related?

IM1.3.8 Describe symmetries of geometric solids.

> **Example:** Describe the rotation and reflection symmetries of a square-based pyramid.

Standard 4

Data Analysis and Statistics

Students find measures of the center and variability of a set of data, as well as construct and analyze data displays and plot least square regression lines.

IM1.4.1 Construct a line plot.

> **Example:** The number of people living on American farms has been decreasing. Construct a line plot for the data, given the number of Americans (in millions) living on farms: (1940, 30.5) (1950, 23.0) (1960, 15.6) (1970, 9.7) (1980, 7.2)

IM1.4.2 Find measures of central tendency for a set of data.

> **Example:** The following gives the number of home runs hit by Roger Maris in his 10 years as a New York Yankee: 13, 23, 26, 16, 33, 61, 28, 39, 14, 8. Determine the mean, median, and mode of this data.

IM1.4.3 Find skewness and symmetry from a graph of data.

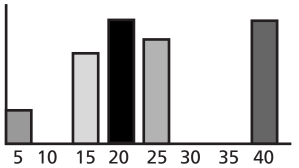
> **Example:** Discuss the skewness of this graph.

IM1.4.4 Construct a histogram using a graphing calculator.

> **Example:** The following gives the pulse rates of 20 students: 55, 95, 62, 94, 93, 91, 64, 67, 80, 80, 82, 70, 72, 76, 88, 84, 88, 86, 78, 78. Make a histogram using a graphing calculator.

IM1.4.5 **Identify clusters, gaps, and outliers for a set of data.**

Example: Describe gaps for the shown data.



IM1.4.6 **Find a linear transformation.**

Example: Consider the following data: 6, 4, 4, 6, 8, 10, 2, 5, 9. Suppose that 5 is added to each value. Compare the mean and average mean deviation of the original and new data sets.

IM1.4.7 **Construct a stem-and-leaf plot using a graphing calculator.**

Example: The following gives the pulse rates of 20 students: 55, 95, 62, 94, 93, 91, 64, 67, 80, 80, 82, 70, 72, 76, 88, 84, 88, 86, 78, 78. Construct a stem-and-leaf plot.

IM1.4.8 **Find the mean absolute deviation for a set of data.**

Example: The following gives the number of keys carried by six students: 3, 3, 6, 0, 4, 4. Calculate the mean, then determine the absolute deviation of each value and the mean absolute deviation.

IM1.4.9 **Find the standard deviation and describe its properties.**

Example: The following gives the number of keys carried by six students: 3, 3, 6, 0, 4, 4. Calculate the mean. Next, determine the square of the difference between the mean and each value and then determine the standard deviation.

IM1.4.10 **Construct a frequency table for a set of data.**

Example: The following gives the pulse rates of 20 students: 55, 95, 62, 94, 93, 91, 64, 67, 80, 80, 82, 70, 72, 76, 88, 84, 88, 86, 78, 78. Make a frequency table with a first class (interval) of 55-59.

IM1.4.11 **Summarize and interpret sets of data using center and variability.**

Example: The following gives the pulse rates of 20 students: 55, 95, 62, 94, 93, 91, 64, 67, 80, 80, 82, 70, 72, 76, 88, 84, 88, 86, 78, 78. Find the mean, range, quartiles, and interquartile range.

IM1.4.12 **Construct a scatterplot from a set of data.**

Example: The following gives data concerning (age in months, height in centimeters) of a child: (36, 86) (48, 90) (51, 91) (54, 93) (57, 94) (60, 95). Construct a scatterplot for this data.

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IM1.4.13 **Calculate the sum of squared differences for a set of data.**

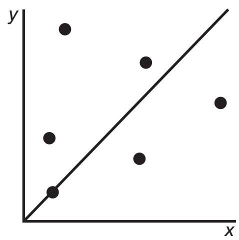
Example: The following gives data concerning (age in months, height in centimeters) of a child: (36, 86) (48, 90) (51, 91) (54, 93) (57, 94) (60, 95). The least squares regression line for these data is $h = 72.0 + .4a$. For each data point, calculate the square of the difference between the observed and predicted values. Calculate the sum of these values.

IM1.4.14 **Plot the least square regression line from a set of data.**

Example: The following gives data concerning (age in months, height in centimeters) of a child: (36, 86) (48, 90) (51, 91) (54, 93) (57, 94) (60, 95). Use a graphing calculator to determine the equation of the least squares regression line.

IM1.4.15 Compare sets of data using scatterplots and the line $y = x$, and interpret these comparisons for real-world data.

Example: Six students were ranked on an English test (x) and a mathematics test (y). The results along with the line $y = x$ are shown on the scatterplot. How many students scored higher in mathematics than English?



IM1.4.16 Recognize patterns in tables and graphs that are modeled by linear equations.

Example: Write a "y = equation" for the following table of values.

X	Y
0	50
1	46
2	42
3	38

Standard 5

Probability

Students use simulations, find probabilities, and use the Law of Large Numbers.

IM1.5.1 Design and use simulations in order to estimate answers related to probability.

Example: A student is taking a true/false test. He is rolling a die to choose an answer. Assume that true and false are both equally likely to be correct answers. Explain how to use a single die to choose a true or false answer.

IM1.5.2 Use empirical (experimental) and theoretical probabilities.

Example: A student flipped two coins. The result was two tails. Find the theoretical probability of flipping two coins and getting two tails. Explain your answer.

IM1.5.3 Understand independent events.

Example: The experiment is rolling a single die. Event A is rolling an even number. Event B is rolling a number greater than three. Are event A and event B independent events? Give an event C that is independent of event B.

IM1.5.4 Use the Law of Large Numbers to understand situations involving chance.

Example: A class is flipping coins to study probability. In one experiment, the coin was flipped 1,000 times. The next day the coin was flipped 2,000 times. Which experiment — the 1,000 flips or the 2,000 flips — has the highest probability of getting 50 percent heads? Why?

IM1.5.5 Understand the concept of a probability distribution. Understand how an approximate probability can be constructed using simulation involving chance.

Example: A class of 25 students is conducting a probability experiment. All the students are standing. Each student flips a coin. If the result is heads, the student sits down. How many students are expected to be standing after the first toss? How many flips are expected until all of the students are seated?

Standard 6

Discrete Mathematics

Students construct graphs, explore algorithms, and use recursion equations and matrices.

IM1.6.1 Construct vertex-edge graph models involving relationships among a finite number of elements.

Example: Bedford High School has the following committee membership.

Executive — Lehman, Smith, Rupp, George
Academic — Smith, Rupp, Vorndran
Extracurriculars — Yoder, Spring
Social — Yoder, Rupp, Jackson
Homecoming — Spring, Marshall, Simpson

Make a vertex-edge graph. Represent the committees with vertices. If two committees share a person, then connect the vertices with an edge. What is the minimum number of meeting times needed so everyone can attend his or her committees' meetings?

IM1.6.2 Construct digraphs.

Example: An athletic conference with four teams has just completed competition. The following matrix shows which team won in head-to-head competition. The "1" in row 1 column 2 means team A won against team B. Use the matrix information to make a digraph showing the same information.

	A	B	C	D
A	0	1	1	0
B	0	0	1	1
C	0	0	0	0
D	1	0	1	0

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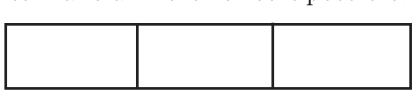
IM1.6.3 Use Euler paths and circuits to solve real-world problems.

Example: The diagram shows a three-block area in a city. A mail carrier must deliver mail in this three-block area. Find a delivery route that is an Euler circuit or modify the diagram to make an Euler circuit possible.



IM1.6.4 Develop the skill of algorithmic problem solving: designing, using, and analyzing systematic procedures for problem solving.

Example: The formula $B = (L/16)(D^2 - 8D + 16)$ describes the number of board feet (B) in a log with a diameter in D in inches and length L in feet. Find the number of board feet in a log with a diameter of 8 inches and having a length of 6 feet.

IM1.6.5 Use a recursion function to describe an exponential function.

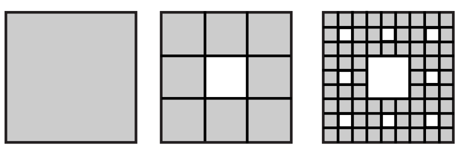
Example: You have just cut your finger with a knife and 10 bacteria cells have entered the wound. This strain of bacteria doubles every 30 minutes. Model the growth of this bacteria using a recursive function. Provide a table of values for the first five hours of the growth of the bacteria.

IM1.6.6 Use a variety of recursion equations to describe a function.

Example: You have saved $1,000 from summer work and have placed it in a savings account at the local bank. This account earns 5 percent compounded annually. You plan to add $1,000 at the end of each year. Model this investment with a recursive function.

IM1.6.7 Use a recursion function to describe a fractal.

Example: Sierpinski's carpet is a fractal pattern. Starting with a solid square "carpet" one meter on a side, smaller and smaller squares are removed from the carpet. The first two stages in forming carpet are shown below. Find recursive and function formulas for the sequence of carpet are that remains at each stage.



Stage 0 Stage 1 Stage 2

IM1.6.8 Use an adjacency matrix to describe a vertex-edge graph.

Example: Use an adjacency matrix to represent the vertex-edge graph from indicator 6.1.

IM1.6.9 Perform row and column sums for matrix equations.

Example: The Acme construction firm builds one-story and two-story houses. The expenses for fees, labor, and materials are shown in the following matrix. Determine the row and column sums of the matrix and explain the meaning of each sum.

	Fees	Materials	Labor
One-Story House	$2,000	$75,000	$25,000
Two-Story House	$2,500	$100,000	$35,000

Standard 7

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems and develop and evaluate mathematical arguments and proofs.

IM1.7.1 Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Example: Fran has scored 16, 23, and 30 points in her last three games. How many points must she score in the next game so that her four-game average does not fall below 20 points?

IM1.7.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the example from indicator 7.1 is 10 points. Is his answer reasonable? Why or why not?

IM1.7.3 Use the properties of the real number system and the order of operations to justify the steps of simplifying functions and solving equations.

Example: Given an argument (such as $3x + 7 > 5x + 1$, and therefore $-2x > -6$, and therefore $x > 3$), provide a visual presentation of a step-by-step check, highlighting any errors in the argument.

IM1.7.4 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: Try "solving" the equations $x + 3y = 5$ and $5x + 15y = 25$ simultaneously. Explain what went wrong.

IM1.7.5 Decide whether a given algebraic statement is true always, sometimes, or never (statements involving linear or quadratic expressions, equations, and inequalities).

Example: Is the statement $x^2 - 5x + 2 = x^2 + 5x + 2$ true for all x, for some x, or for no x? Explain your answer.

IM1.7.6 Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

Example: What type of reasoning are you using when you look for a pattern?

IM1.7.7 Use counterexamples to show that statements are false, recognizing that a single counterexample is sufficient to prove a general statement false.

Example: Use the demonstration-graphing calculator on an overhead projector to produce an example showing that this statement is false: all quadratic equations have two different solutions.